Theory of an Efficient Electronic Phase Shifter Employing a Multilayer Dielectric-Waveguide Structure

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***Abstract*—Multilayer dielectric-waveguide theory is applied to the design of a submillimeter phase shifter driven by a p-i-n diode. In a three-layer structure, proper choice of layer thicknesses can yield a predicted phase shift/unit length about ten times that possible from a single slab at the same frequency. In a properly specified four-layer structure, still larger shifts, limited primarily by the required excursion in effective refractive index, are shown to be possible. A further advantage of the four-layer structure is that it can be designed to have a very thin active region, thus lowering diode power/unit phase shift, which is proportional to the cross section of the intrinsic region of the p-i-n diode.**

I. Introduction

IN DEVICES for the submillimeter-wavelength region of the electromagnetic spectrum, design approaches analogous to those of integrated optics [1], [2] appear attractive. These techniques have been used by Jacobs and Chrepta [3] to design an electronic phase shifter for millimeter wavelengths. The mechanism of operation of their device is the electronic alteration of the refractive index in the intrinsic region of a p-i-n diode. They obtained (experimentally measured) phase shifts/unit length, $\Delta\phi/\Delta l$ of the order 50^0/cm with Si waveguide in the wavelength range 0.42–1.8 cm.

Prospects for direct extension of these designs to the far-IR submillimeter (25 μm–1 mm) wavelength range are not promising. The principal reason is that the refractive-index change due to injected carriers at a constant injection current is given by [4]

$$\Delta n = \frac{-Ne^2}{2nm_e{}^*\varepsilon_0\omega^2} \tag{1}$$

where N is the injected carrier concentration, n is the unperturbed refractive index, $m_e{}^*$ is the optical effective mass of the carrier, ε_0 is the permittivity of vacuum, and ω is the angular frequency. The ω^{-2} dependence in (1) drastically reduces the magnitude of the effect at the shorter wavelengths of interest here.

Recently, the idea of using the waveguide structure itself to enhance phase modulation has been addressed in the integrated-optics literature [5], [6]. These papers use as examples electrooptic layers for the active regions of their waveguides, but the theories presented actually treat the transformation of *any* changes Δn in the bulk refractive index into a larger change $\Delta \boldsymbol{n}$ in the effective refractive index $\boldsymbol{n}$ of the guided mode. This figure of merit for the optical region $\Delta \boldsymbol{n}$ is related to that used in the submillimeter literature $\Delta\phi/\Delta l$ by

$$\frac{\Delta\phi}{\Delta l} = \frac{2\pi}{\lambda_0}\Delta \boldsymbol{n} \tag{2}$$

where λ_0 is the vacuum wavelength. From (1) and (2), if a device for lower frequencies is just scaled down for operation at higher ones, with structure and materials kept constant, $\Delta\phi/\Delta l$ will be proportional to ω^{-1}.

Another important parameter is the required drive power to operate the phase shifter at a certain modulation frequency. For a given method and frequency of modulation, and for a prescribed phase shift, this power is increased with the volume of the active region over which n changes. Hence devices with smaller active-medium volumes are preferred over those with large ones. Lotspeich's [5] analysis of a single-active slab waveguide predicted $\Delta \boldsymbol{n}/\Delta n \lesssim 3$, but obtaining $\Delta \boldsymbol{n}/\Delta n \approx 3$ required slabs of the order of 100 λ_0 thick. For the three-layer structure of [6], $\Delta \boldsymbol{n}/\Delta n \approx$ 10–15 is predicted, with required active-layer thicknesses of the order 5–25 λ_0. Layer thickness tolerances for this structure are somewhat tight [6]; but since they are proportional to wavelength, they will be a lesser problem in this wavelength range than in the visible.

In this paper two structures, one of three layers and one of four, are described and analyzed as electronic phase shifters for the submillimeter region. The driving refractive-index change Δn is assumed to be obtained by carrier injection, as in [3]. However, under the conditions of field orientation, electrooptic crystal symmetry, and mode polarization described in [6], the values of $\Delta \boldsymbol{n}/\Delta n$ predicted below would be valid for electrooptic devices also.

II. Theory

For the devices to be studied, the waveguide cross-sectional dimension perpendicular to the layers will be much less than that parallel to them. Hence the guided modes can be assumed to be TE-like or TM-like. Multilayer slab-waveguide theory in matrix form [6] can then be used to calculate an effective refractive index for the guided mode in the center of the waveguide, and in the regions next to the center, but in the plane of the device. Knox and Toulious [7] and McLevige [8] have shown that mode confinement parallel to the device plane can then be treated by solving a waveguide equation in which these effective refractive indices are assumed homogeneous perpendicular to the

Manuscript received November 16, 1976. This work was supported by the U.S. Office of Naval Research.

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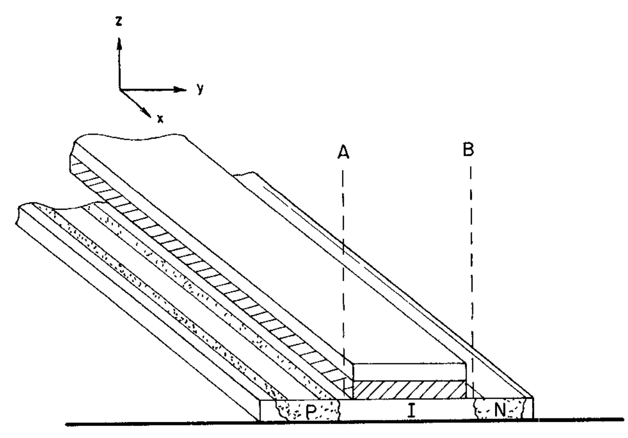

Fig. 1. Three-layer waveguide phase shifter driven by p-i-n diode.

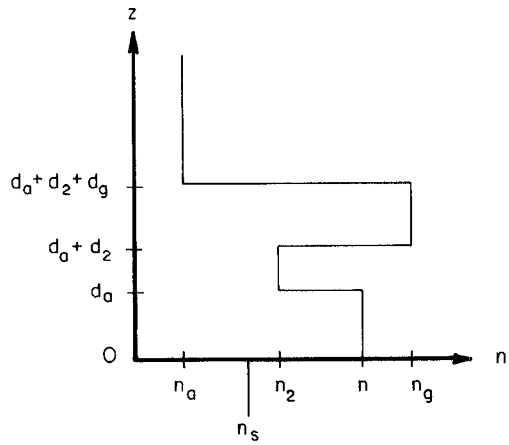

Fig. 2. Refractive index versus z in the central region (x-z plane) of Fig. 1. The x-y plane is the substrate surface.

device plane, and having thicknesses corresponding to the actual device dimensions parallel to the device plane. Under these conditions, the effective refractive index for the device is approximately that calculated for a multilayer slab waveguide without lateral mode confinement.

Even if $\boldsymbol{n}$ for the laterally delineated guide is not the same as for the slab waveguide, it is reasonable to expect maxima in $|\Delta \boldsymbol{n}/\Delta n|$ to occur under the same conditions for the delineated guide as for the slab.

With these approximations, the modal equation for the dielectric multilayer slab is [1], [6]

$$4\pi n_g\sqrt{1-q^2}\,\frac{d_g}{\lambda_0} = \phi_1(q) + \phi_2(q) + 2m\pi, \qquad m = 0,1,2\cdots \tag{3}$$

where n_g and d_g are, respectively, the refractive index and thickness of layer g (chosen arbitrarily), $q = \boldsymbol{n}/n_g$, and ϕ_1, ϕ_2 are the effective phase shifts [6] on total internal reflection at the boundaries of medium g. Equation (3) can be solved for q to yield $\boldsymbol{n}$ if λ_0 and all indices and thicknesses are known. Following [6], the ratio which gives the effect of the multilayer structure on small changes in $\boldsymbol{n}$ is

$$\frac{d\boldsymbol{n}}{dn} = \frac{-n_g\,\partial(\phi_1+\phi_2)/\partial n + (\phi_1+\phi_2)\delta}{\dfrac{q_0(\phi_1+\phi_2)}{1-q_0^2} + \dfrac{\partial(\phi_1+\phi_2)}{\partial q}} + q_0\delta \tag{4}$$

where q_0 is a solution of (2), and $\delta = 1$ if medium g is the active medium and $\delta = 0$ otherwise.

Estimating $|\Delta n|$ by (1), using $n = 3.5$, m_e^* equals free electron mass, and $\omega = 1.89 \times 10^{13}\ \mathrm{s}^{-1}$ ($\lambda_0 = 0.1$ mm) gives $|\Delta n| \approx 1.3 \times 10^{-4}$ for an injected carrier concentration of $10^{14}\ \mathrm{cm}^{-3}$. From (1), $|\Delta n|$ is linear in injected carrier concentration. For refractive-index changes of this and lesser magnitude, $\Delta\boldsymbol{n}/\Delta n \approx d\boldsymbol{n}/dn$, and (4) accurately gives the phase-shift enhancement due to the guide structure.

III. Three-Layer Phase Shifter

The three-layer phase shifter is shown in Fig. 1. The p-i-n diode is fabricated in a narrow strip geometry, with the two additional dielectric strips deposited between the p- and n-regions on the diode layer. Metal contacts connect the p- and n-regions to the drive circuit. The refractive-index profile of this structure in the waveguiding region (between A and B in Fig. 1) is shown in Fig. 2. The values of the indices chosen do not affect the upper limit of $\sim n_g^2$ on $\Delta\boldsymbol{n}/\Delta n$, but the relative inequalities of index shown in Fig. 2 must be preserved for this theory to be valid. The guided wave is confined (roughly) to the region between A and B, since this device is essentially a dielectric stripline [9], [10]. Hence metallic absorption of the guided wave can be kept very small and will be neglected.

According to [6], q_0 should be chosen slightly less than n/n_g for large $|\Delta\boldsymbol{n}/\Delta n|$. The required thicknesses d_a and d_g are then found from (TE modes)

$$\frac{d_a}{\lambda_0} = \frac{1}{2\pi\sqrt{n^2 - n_g^2 q_0^2}} \cdot \left(\tan^{-1}\left[\frac{n_g^2 q_0^2 - n_2^2}{n^2 - n_g^2 q_0^2}\right]^{1/2} + \tan^{-1}\left[\frac{n_g^2 q_0^2 - n_s^2}{n^2 - n_g^2 q_0^2}\right]^{1/2} + j\pi\right) \tag{5}$$

where $j = 0,1,2,3,\cdots$, and (3) with $q = q_0$.

The index range $3 \lesssim n \lesssim 4$ appears appropriate for the submillimeter region. Accordingly, we consider as an example a three-layer structure, as in Figs. 1 and 2, with $n_g = 3.90$, $n_2 = 3.70$, $n = 3.80$, and $n_a = n_s = 1.00$ (air). The design value for the operating point q_0 is chosen as 0.9742, slightly lower than $q = 0.9744$, where the active layer becomes cut off. Then, for $j = 0$ (lowest order mode in the active layer) (5) yields $d_a/\lambda_0 = 7.06$ and (3) yields $d_g/\lambda_0 = 0.1012$. For $q_0 \approx n/n_g$, as is true here, d_2/λ_0 is not critical [6], but need only be small enough so that coupling between the two high-index layers is fairly strong. Using (4) gives $d\boldsymbol{n}/dn \approx 9.4$ over a range of n corresponding to $\Delta n \approx 10^{-4}$, the refractive-index change expected from carrier injection. Using (2) and a wavelength $\lambda_0 = 0.1$ mm gives a predicted $\Delta\phi/\Delta l$ of 34°/cm, if $\Delta n = 10^{-4}$, or roughly ten times the phase shift of a scaled-down single-layer device.

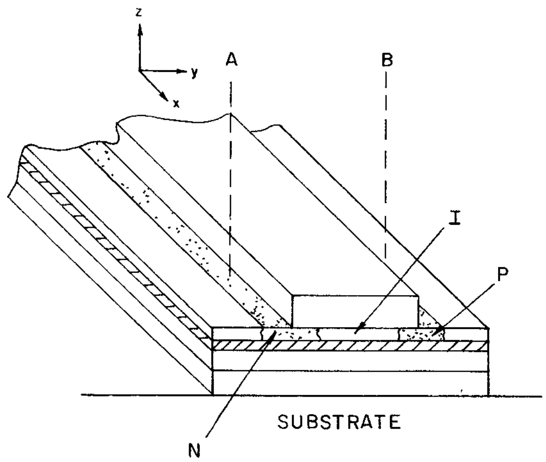

Fig. 3. Four-layer waveguide phase shifter driven by a thin p-i-n structure in the top layer. The narrower block of material atop the structure is not part of the waveguide, but serves only to provide an $n_a \neq 1$.

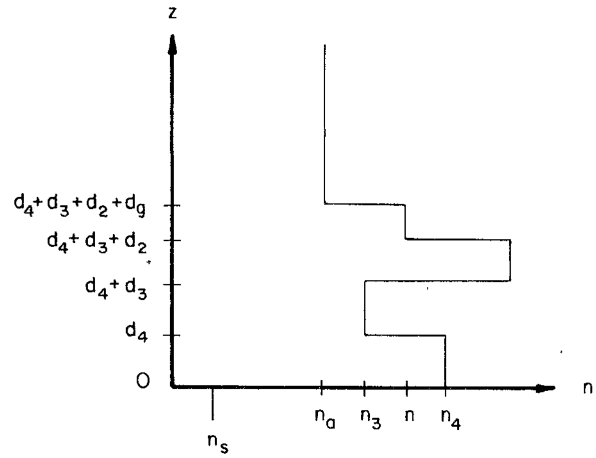

Fig. 4. Refractive index versus z in the x-z plane of Fig. 3. The x-y plane is the substrate surface.

IV. Four-Layer Phase Shifter

The four-layer phase shifter, shown in Fig. 3, offers two advantages over the three-layer device. First, ratios of $|dn/dn| > n_g^2$ are possible. Secondly, it can be designed to make use of a very thin active layer, hence requiring less p-i-n-diode drive current for a given injected-carrier density. Mode confinement is obtained here through free-carrier dispersion in the p-i-n-diode structure itself, since the p- and n-regions have lower refractive indices. The refractive-index profile in the waveguiding region (the intrinsic region in the diode between A and B in Fig. 3) is shown in Fig. 4. As in the three-layer case, the values of the indices are not important, but the relative inequalities in Fig. 4 must be preserved for the following theory to be valid.

Equations (3) and (4) are still valid for this structure. However, ϕ_2 and its derivative take different forms. For values of q_0 near n_3/n_g, it can be shown [11] that ϕ_2 and $\partial\phi_2/\partial q$ are given by (TE modes)

$$\phi_2 = \tan^{-1}\left\{\frac{(1 - r_{g2}^2)\sin\theta}{2r_{g2} + (1 + r_{g2}^2)\cos\theta}\right\} \tag{6}$$

and

$$\frac{\partial\phi_2}{\partial q} = \frac{(r_{g2}^2 + 2_{g2}\cos\theta + 1)\left\{(1 - r_{g2}^2)C_0\left(\frac{d_2}{\lambda_0}\right) - 2\sin\theta\,\frac{\partial r_{g2}}{\partial q}\right\}}{[2r_{g2} + (1 + r_{g2}^2)\cos\theta]^2 + [(1 - r_{g2}^2)\sin\theta]^2} \tag{7}$$

where r_{g2} is the Fresnel coefficient for the interface at $z = d_4 + d_3 + d_2$ in Fig. 4, and

$$\theta = \phi_{23} - \frac{4\pi\, d_2}{\lambda_0}\sqrt{n_2^2 - n_2^3} \tag{8}$$

$$C_0 = \frac{4\pi n_3/n_g}{\sqrt{n_2^2 - n_3^2}} \tag{9}$$

and ϕ_{23} is the effective phase shift on total internal reflection at $z = d_4 + d_3$ in Fig. 4, calculated using the matrix technique of [6]. From (7), we note that proper choice of d_2 and ϕ_{23} will make $\partial\phi_2/\partial q < 0$ for $q_0 \approx n_3/n_g$. If $\partial\phi_2/\partial q$ is sufficiently negative, it will partially offset the other terms in the denominator of the first term in (4), resulting in a large value for $|dn/dn|$. Criteria for the existence of one or more sets of layer thicknesses which result in a small denominator of the first term in (4) are discussed in [11]. It is sufficient for our purpose here to illustrate how this condition can be used to achieve a $|dn/dn| > n_g^2$ when the active layer, in this case medium g, is thin.

As an example, suppose $n_g = 3.50$, $n_2 = 3.80$, $n_3 = 3.40$, $n_4 = 3.60$, $n_s = 1.0$ (air); and $n_a = 3.30$. There is no unique design procedure or set of thicknesses which give both $q_0 = n_3/n_g$ and a large $|dn/dn|$. Since, for a p-i-n-diode active medium, a small d_g is desirable, $\phi_1 + \phi_2$ should be small and positive, which implies $\phi_2 < 0$. Accordingly, θ will be held constant at $-\pi/2$ in our design procedure, and ϕ_2 will be constant as well. For the single interface at $z = d_4 + d_3 + d_2 + d_g$ in Fig. 4, ϕ_1, and $\partial\phi_1/\partial q$ are related by (at $q = q_0 = n_3/n_g$)

$$\frac{\partial\phi_1}{\partial q} = \frac{2n_3\cot(\phi_1/2)}{n_g^2 - n_3^2} \tag{10}$$

with ϕ_1 determined by n_a.

As an example, we consider the case where $\phi_{23} = -\pi/6$ with $n_a = 3.3$, $n_g = 3.5$, $n_2 = 3.8$, $n_3 = 3.4$, $n_4 = 3.6$, $n_s = 1.0$ (air). Using these quantities in (6)–(9), and requiring $q_0 = n_3/n_g$, results in $d_g/\lambda_0 = 0.0201$, $d_2/\lambda_0 = 0.0490$, $d_3/\lambda_0 = 0.2875$, $d_4/\lambda_0 = 0.352$, and $dn/dn = -20.9$. For $\Delta n \approx 10^{-4}$ and $\lambda_0 = 0.1$ mm, this represents a $\Delta\phi/\Delta l$ of 75°/cm.

It is necessary here to point out that (6)–(9) are accurate only for values of q within $\approx 10^{-2}$ of n_3/n_g [11]. Hence arbitrarily large $\Delta\phi/\Delta l$ cannot be obtained by reducing the denominator of the first term of (4) to nearly zero, since small Δn would take n out of the range over which (7)–(10) are valid. However, for the case of interest here, where $\Delta n \approx 10^{-4}$ and $|dn/dn| \lesssim 50$, this analysis is valid over the entire accessible range of n.

V. Power Requirements

The power requirements of the electrooptic phase modulators used in integrated optics are often expressed as power per unit bandwidth, $P/\Delta f$, since no power is required to maintain a time-independent phase shift. For a p-i-n diode, forward current is proportional to the product of intrinsic region volume and injected-carrier concentration, and drive power is proportional to forward current [12]. Thus a more appropriate expression for the p-i-n device would be $P/\Delta\phi$, power per unit phase shift, which depends on active region cross section S, and $|dn/dn|$ as

$$\frac{P}{\Delta\phi} \propto \frac{S}{|dn/dn|}. \tag{11}$$

The four-layer device previously described reduces S and increases $|dn/dn|$ in comparison with a single-layer phase shifter, while the three-layer device relies primarily on increased $|dn/dn|$.

VI. Conclusion

We have analyzed the operation of three- and four-layer electronic phase shifters for the submillimeter-wavelength region, driven by p-i-n diodes. The multilayer dielectric structures studied can be designed to compensate for the decrease in free-carrier dispersion at these shorter wavelengths. Phase shifts/unit length of the same order as those possible at centimeter wavelengths, and up to 20 times those possible without multilayer structures at submillimeter wavelengths, are predicted, along with lower drive power requirements.

The devices studied in this paper have not been optimized. Hence still higher values for $\Delta\phi/\Delta l$ and lower ones for $P/\Delta\phi$ appear to be possible.

Both devices studied take advantage of resonancelike effects at certain values of q to obtain the desired results. Hence they will be highly frequency dependent, since all thicknesses in the theory are expressed in terms of the vacuum wavelength λ_0. Efficient prefiltering, or the use of single-frequency sources, may be required in device applications.

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